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Design and Analysis of Double Wishbone Suspension

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ABSTRACT

Now a day's all types of light trucks and passenger cars use independent front suspensions, because of the betterresistance to vibrations. One of the commonly used independent front suspension system is referred as doublewishbone suspension.Thestudy of suspension system and dynamic analysis are discussed in this paper.The links of the suspension are assumed to be flexible and elastic.Stiffness, mass, and geometric stiffness matricesare obtained by using Finite Element Method. In order to express the linear equation of motion, suspension link forcesrequired for the geometric stiffness matrices are assumed as constant. Also, the oscillations of the suspension links areneglected since the base displacement is chosen in small amplitude. The FEA was done by dividing the lower and the upperarms into two elements. Double wishbone suspension system of a quarter cars is modeled assuming the suspension links to berigid.

Keywords- ***Double wishbone suspension system, Finite Element Analysis, Modeling, Dynamic Analysis***

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I. INTRODUCTION

aNow a day's nearly all types of passenger cars and light trucks use independent front suspensions, because of the better resistance to vibrations. One of the commonly used independent front suspension system is referred as double wishbone suspension. The quarter car with the double wishbone suspension system is modeled for two different approaches to the suspension links as rigid and flexible. Therefore, the dynamic analyses of these models are investigated by the finite element method. This paper deals with the analytical method and the finite element method. The element stiffness, the mass and the geometric matrices are explained for the plane frame element respectively. Modeling of double wishbone suspension is presented in two models. Vibrations of the double wishbone suspension system, natural frequencies and response to base excitation are studied.

In Finite Element Method, a complex region defining a continuum is discredited into simple Geometric shapes called finite elements. The material properties and the governing relationships are considered over
these elements and expressed in terms of unknown values at the nodes. An assembly process, duly considering the loading and constraints, results in a set of equations. Solution of these equations gives us approximate behavior of the continuum use of the analytical method and finite element method for the dynamic analysis of double wish bone suspension system is described. Mass, stiffness and geometric stiffness matrices are derived. The plane frame element is selected to model the double wishbone suspension member.

II. CHARACTERISTIC MATRICES OF THE PLANE FRAME ELEMENT

A planar (2-D) frame element is subjected to both axial and bending deformations. Therefore, the plane frame

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element has three degrees of freedom per node together with local displacements (u 1, v1 and θ1) and global displacements (ū 1, v1 and θ1) as shown in Figure 1.1. The nodal displacement vector is given by

$$\{q\}= \{u_1\, v_1\, \theta_1\, u_2\, v_2\, \theta_2\}^T \quad (1.1)$$

The element stiffness matrix for a 2-D frame element can be constructed by superimposing both axial and bendingstiffness. The element kinetic and strain energy functions for plane frame element are given in terms of localcoordinates, as follows; given in terms of local coordinates as follows;

$$T = \frac{1}{2}\int_e \rho A\, (u^{-2}+ v^{-2})\, dx \quad (1.2)$$

Where the over dot shows the differentiation with respect to time.

$$U = \int_e EA\left(\frac{du}{dx}\right)^2 dx + \frac{1}{2}\int_e EI\left(\frac{d^2u}{dx}\right) dx \quad (1.3)$$

$$U = [\, Nu\, (\xi)]\, \{u\}_e \quad (1.4a)$$

$$v = [\, N\, v\, (\xi)]\, \{v\}e \quad (1.4b)$$

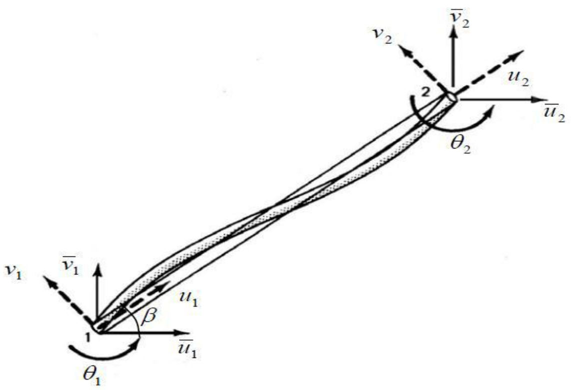

Figure 1.1. Plane frame element
(Source: Belegundu and Chandrupat la 1997)

III. ELASTIC STIFFNESS MATRIXES

Substituting the displacement functions given in Equation (1.3) into strain energy expression given in Equation (1.4)gives,

$$T = \frac{1}{2}\{\bar{u}\}_e^T[m]_e\{\bar{u}\}_e \quad (1.5)$$

$$[\tilde{k}]_e = \frac{EI_z}{2a^2}\begin{pmatrix} (\frac{a}{r_2})^2 & 0 & 0 & -(\frac{a}{r_2})^2 & 0 & 0 \\ 0 & 3 & 3a & 0 & 3a & -3a \\ 0 & 3a & 4a^2 & 0 & -3a & 2a^2 \\ & 0 & 0 & (\frac{a}{r_2})^2 & 0 & 0 \\ 0 & -3 & -3a & 0 & 3 & -3a \\ 0 & 3a & 2a^2 & 0 & -3a & 4a^2 \end{pmatrix} \quad (1.6)$$

Where, $l_e = 2a$ and $r_z^{\,2} = I_z / A$ [10]

IV.GEOMETRIC STIFFNESS MATRIX

The geometric strain energy in the element is

$$U_g = \int_0^L \frac{P}{2}\left(\frac{du}{dx}\right)^2 dx\, \frac{1}{2} = \{q\}[s]_e\{q\} \quad (1.7)$$

The geometric stiffness matrix [s] for a plane frame is developed from the Equation (1.7)

$$[\tilde{s}]_e = \frac{P}{60_a}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 36 & 6a & 0 & -36 & 6a \\ 0 & 6a & 16a^2 & 0 & -6a & -4a^2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -36 & -36 & 0 & 36 & -6a \\ 0 & 6a & 6a & 0 & -6a & 16a^2 \end{pmatrix} \quad (1.8)$$

Where, le = 2a and P is the axial force [4]

V.MASS MATRIXES

Substituting the displacement functions given in Equation (1.4) into the kinetic energy expression given in Equation(1.2) gives

$$T = \frac{1}{2}\{\bar{u}\}_e^T[m]_e\{\bar{u}\}_e \quad (1.9)$$

Where

$$[\tilde{m}]_e = \frac{\rho Aa}{105}\begin{pmatrix} 70s & 0 & 0 & 35 & 0 & 0 \\ 0 & 78 & 22a & 0 & 27 & -13a \\ 0 & 22a & 8a^2 & 0 & 13a & -6a^2 \\ 35 & 0 & 0 & 70 & 0 & 0 \\ 0 & 27 & 13a & 0 & 78 & -22a \\ 0 & -13a & -6a^2 & 0 & -22a & 8a^2 \end{pmatrix}$$

(1.10)

In which le = 2a, ρ is the mass per unit volume, and A is the cross -sectional area of each element [10]

VI.STIFFNESS OF THE SPRING

In finite element model, stiffness of helical-shaped springs used in suspension system may be expressed in matrix notation considering the plane frame element displacement vector as follows;

$$[\tilde{k}_s] = k\begin{pmatrix} 70 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad (1.11)$$

Where k is the stiffness coefficient of the spring by the equation;

$$k = \frac{G_s\, d4}{64n\, R_s^{\,3}} \quad (1.12)$$

Where,

The stiffness is a function of the shear modulus (Gs), the diameter of the turns of coils (Rs), the diameter of the coils (d), and the number of the coils (n) [14]

A. Page Layout

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Your paper must use a page size corresponding to A4 which is 210mm (8.27") wide and 297mm (11.69") long. The marginsmust be set as follows:

- Top = 19mm (0.75")
- Bottom = 43mm (0.75")
- Left = Right = 14.32mm (0.56")

Your paper must be in two column format with a space of 4.22mm (0.17") between columns.

VII.COORDINATE TRANSFORMATIONS

If a frame member is inclined in global coordinate system as shown in Figure 3.1, the element stiffness, mass and geometric stiffness matrices require the planar transformation. Figure 3.1 shows the nodal freedoms in local and global systems. The relation between the local and global displacements is

$$\begin{Bmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \end{Bmatrix} \begin{pmatrix} c & s & 0 & 0 & 0 & 0 \\ -s & c & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & c & s & 0 \\ 0 & -1 & 0 & -s & c & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \end{Bmatrix}$$

Where c = cosβ and s = sinβ

In the short notation, Equation (1.13) can be written as

$$\{u\}_e = [R]_e\{u\}_e \qquad (1.14)$$

Substituting Equation (1.14) into the energy expressions given in Equations (1.2) and (1.3) gives

$$T = \frac{1}{2}\{\bar{u}\}_e^T[m]_e\{\bar{u}\}_e \quad (1.15)$$

$$U_e = \frac{1}{2}\{\bar{u}\}_e^T[k]_e\{\bar{u}\}_e \quad (1.16)$$

$$U_g = \frac{1}{2}\{\bar{u}\}_e^T[s]_e\{\bar{u}\}_e \quad (1.17)$$

The stiffness and mass matrices for a planar frame element are expressed in terms of the global coordinate system as given below

$$[M]_e = [R]_e^T[m]_e[R]_e \qquad (1.18)$$

$$[K]_e = [R]_e^T[k]_e[R]_e \qquad (1.19)$$

$$[S]_e = [R]_e^T[S]_e[R]_e \qquad (1.20)$$

VIII.MODELLING OF DOUBLE WISHBONE SUSPENSION

MODELLING ASSUMPTIONS

Figure 1.2 shows a part of a chassis with a double wishbone suspension system. The mechanical system consists of amain chassis, a double wishbone suspension subsystem and a wheel. A suspension spring, lower and upper arms areincluded in the suspension sub-system. The lower and upper arms are modeled by simple links the chassis isconstrained to move vertically upward or downward. The wheel can be modeled as a linear translational spring. The motion of the wheel over the road the quarter car with the double wishbone suspension is modeled depending on two different assumptions due to the suspension links. In the first model, the links of the suspension are assumed to be rigid links. In the second model, finite element model, links are modeled to be flexible

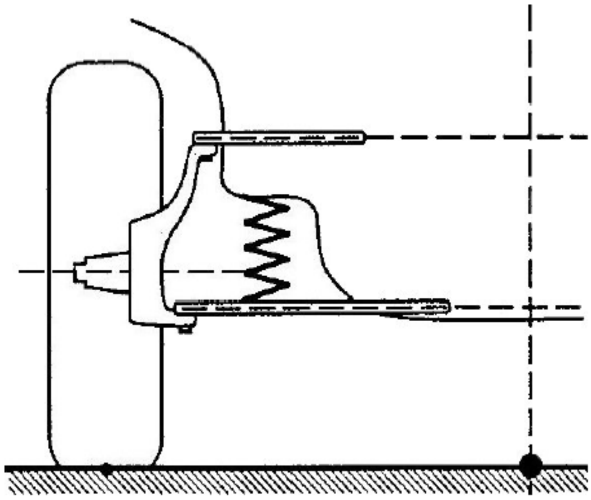

Figure 1.2 A quarter car with the double wishbone suspension (Source:Gillespie 1992)

For analysis purpose, the model of the quarter car with the double wishbone suspension assumed to travel with constant velocity on a road surface characterized by a displacement y (t) g. Angular displacements of the lower and upper linksare negligible since the amplitude of the base displacement (amplitude of y (t) g) is chosen in small amplitude. On the other hand, in order to have the linear equation of motion, axial link forces are assumed as constant.

IX.SIMPLE MODELLING OF SUSPENSION SYSTEM

Double wishbone suspension of a quarter cars is modeled assuming the suspension links to be rigid. The model is shown in Figure 3.3.The mass t m represents approximately the mass of the wheel plus part of the mass of the suspension arms, c m represents approximately 1/4 of the car mass.

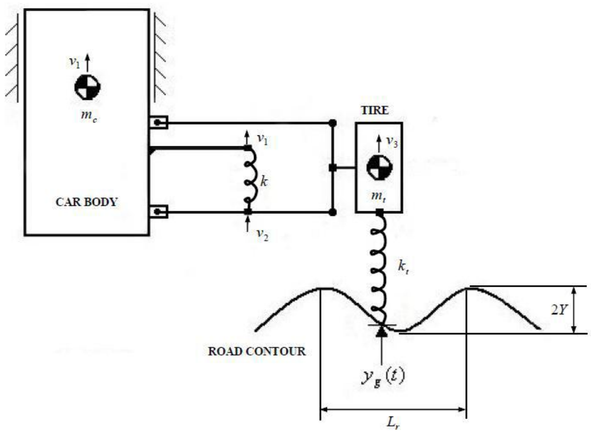

Figure 1.3. Simple model of the suspension system

Simple Modelling Of Suspension System

Double wishbone suspension of a quarter cars is modelled assuming the suspension links to be rigid. The model is shown in Figure 1.4. The mass t m represents approximately the mass of the wheel plus part of the mass of the suspension arms; m_c represents approximately 1/4 of the car mass. The excitation comes from the road irregularity. It is considered that the spring is located in the middle of the lower control arm. The kinetic and strain energies as;

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$$T = \frac{1}{2} m_c v_1^2 + \frac{1}{2} m_t v_3^2 = \frac{1}{2}\{q\}^T [M]\{q\} \quad (1.21)$$

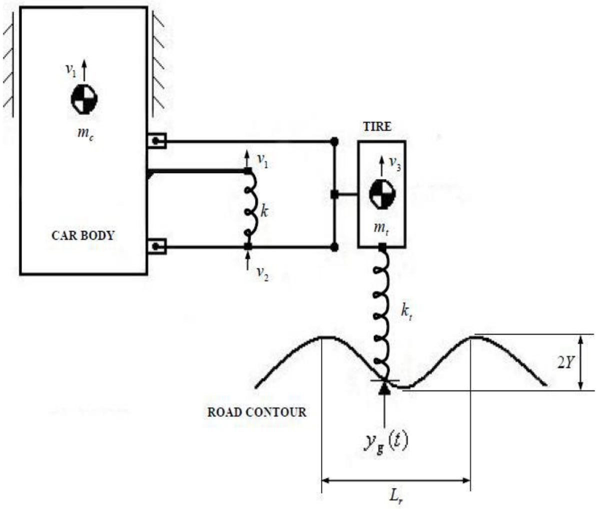

Figure 1.4. Simple model of the suspension system

$$U = \frac{1}{2} k_t (y_g - v_3)^2 + \frac{1}{2} k (y_2 - v_1)^2 = \frac{1}{2}\{q\}^T [K] \{q\}$$
(1.22)

The Lagrange's equations;

$$\frac{d}{dt}\left(\frac{\partial T}{\partial q_i}\right) + \left(\frac{\partial U}{\partial q_i}\right) = Q$$

$$i = 1, 2 \ldots n \quad (1.23)$$

Where the total strain energy U = U_e +, and Q_i are generalized forces. The Lagrange's equations (1.23) yield the equations of motions in matrix form to find the natural frequencies

$$[M]\{q\} + [K]\{q\} = \{Q\} \quad (1.24)$$

The equation (1.23) can be written for v_1

$$\frac{d}{dt}\left(\frac{\partial T}{\partial v_1}\right) + \frac{\partial U}{\partial v_1} = Q_1 \quad (1.25)$$

$$v_2 = \left(\frac{v_1 + v_3}{4}\right) \quad (1.26)$$

$$m_c v + k\left(\frac{v_1 - v_3}{4}\right) = 0 \quad (1.27)$$

The equation (1.23) can be written for v_3

$$\frac{d}{dt}\left(\frac{\partial T}{\partial v_3}\right) + \frac{\partial U}{\partial v_3} = Q_2 \quad (1.28)$$

$$m_t v_3 + k_t (v_3 - y_g) + k\left(\frac{v_1 - v_3}{4}\right) = 0 \quad (1.29)$$

The differential equations in matrix form are

$$\begin{bmatrix} m_c & o \\ o & m_t \end{bmatrix} \begin{Bmatrix} \ddot{v}_1 \\ \ddot{v}_3 \end{Bmatrix} + \begin{bmatrix} k/4 & -k/4 \\ -k/4 & (k/4) + k_t \end{bmatrix} \begin{Bmatrix} v_1 \\ v_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ k_t y_g \end{Bmatrix} \quad (1.30)$$

The system characteristic matrices are

$$[M] = \begin{bmatrix} m_c & o \\ o & m_t \end{bmatrix} \quad (1.31)$$

$$[K] = \begin{bmatrix} k/4 & -k/4 \\ -k/4 & (k/4) + k_t \end{bmatrix} \quad (1.32)$$

The generalized force vector is

$$\{Q\} = \begin{Bmatrix} 0 \\ k_t y_g \end{Bmatrix} \quad (1.33)$$

On the other hand, Equation (1.30) represents a mathematical model shown in Figure 1.4.If the base displacement isdefined by a single frequency harmonic of the form as, $y_g(t) = Y \sin w_e t$.

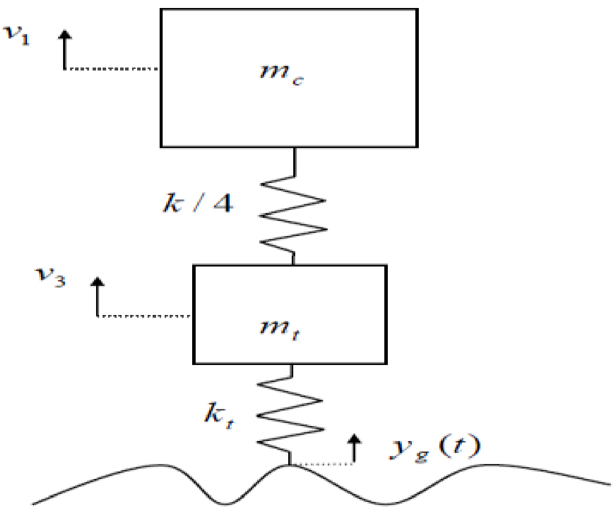

Figure 1.4. A quarter car suspension model
(Source:Gobbi and Mast inug)

The frequency of base motion is w_e

$$w_e = \frac{2\pi v}{L_r} \quad (1.34)$$

Where V is the vehicle speed and Lr is the period of the road profile.

Finite Element Modelling Of Suspension System

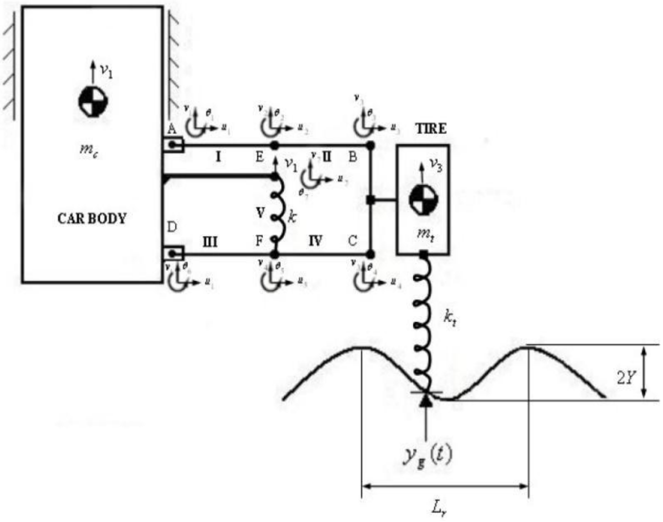

Figure 1.5. Finite element model of the suspension system

The lower and the upper arms are divided into two elements, as shown in Figure 1.5. The degrees of freedom of node I are u_i, $v_i$and θ_i. The degree of freedom vi is transverse displacement and $u_i$is axial displacement and $\theta_i$is slope or rotation.

The global displacement vector

$$\{q\} = \{u_1, v_1, \theta_1, u_2, v_2, \theta_2, u_3, v_3, \theta_3, u_4, \theta_4, u_5, v_5, \theta_5, \theta_6, \theta_7\}^T \quad (1.35)$$

The local degrees of freedom for a single element are represented by Equation (1.1);

$$\{q\}=\{u_1, v_1, \theta_1, u_2, v_2, \theta_2,\}^T \quad (1.36)$$

The connectivity table for the element solution is given in Table 1.1. Every node in an element has both a local coordinate and a global coordinate. The elastic stiffness, geometric stiffness, mass matrices are found from Equations (1.6), (1.8), and (1.10) for each the plane frame element. The global stiffness, geometric, and mass matrices are obtained by assembling these element matrices. The spring element given in Equation (1.12) is considered a frame element.

Element Number	**Local Freedom Numbers**					
	1	**2**	**3**	**4**	**5**	**6**
I.	**1**	**2**	**3**	**4**	**5**	**6**
II.	**4**	**5**	**6**	**7**	**8**	**9**
III.	**1**	**2**	**15**	**12**	**13**	**14**
IV.	**12**	**13**	**14**	**10**	**8**	**11**
V.	**12**	**13**	**14**	**1**	**2**	**16**

Table1.1. Global freedom numbers for the finite element model

X.CONCLUSION

The quarter car with the double wishbone suspension system has been modeled for two different approaches to the suspension links to be rigid and flexible. Therefore, the dynamic analyses of these models have been investigated by the finite element method. Analysis of the results showed that the agreement between the simple model and flexible model without unloaded links is excellent for both natural frequencies and the time responses. Therefore, the simple model is adequate for the first design step. However, in order to obtain the more accurate results, for example natural frequencies and time responses, it is necessary to consider the finite element model of the suspension system.

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